

# Magnetochronology of solar-type star dynamos

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Received 11 September, 2023; accepted 23 January, 2024

## ABSTRACT

**Aims.** In this study, we analyse magnetic field properties of a set of 15 global magnetohydrodynamics simulations of solar-type star dynamos conducted using the ASH code. Our objective is to enhance our understanding of these properties by comparing theoretical results to current observations, aiming finally to provide fresh insights into the field.

**Methods.** We analyse rotational and magnetic properties as a function of various stellar parameters (mass, age, rotation rate) in a *Sun in time* approach in our extended set of 3D MHD simulations. To facilitate direct comparisons with stellar magnetism observations using various Zeeman-effect techniques, we decompose numerical data into vectorial spherical harmonics.

**Results.** The comparison of the trends we found in our simulations set reveals a promising overall agreement with the observational context of stellar magnetism, enabling us to suggest a plausible scenario for the magneto-rotational evolution of solar-type stars. In particular, we find that the magnetic field may reach a minimum in amplitude at a transition value in Rossby number near unity. This may have important consequences on the long term evolution of solar-type stars, by impacting the relation between stellar age, rotation and magnetism. This supports the need for future observational campaigns, especially for stars in the high Rossby number regime.

**Key words.** Sun: magnetic fields – Sun: evolution – Sun: rotation – Dynamo – Stars: activity – Stars: fundamental parameters – Stars: evolution – Stars: solar-type – Stars: rotation – Methods: Numerical – Methods: data analysis

## 1. Introduction

Since the pioneering work of Skumanich (1972), followed by Barnes (2003), there seems to be a link between the age of a solar-type star and its rotation rate. Older stars usually rotate slower than their younger equivalent, this is the well-known gyrochronology proposed by Barnes (2003). This in turn seems to influence the degree of magnetic activity a given solar-type star harbours. Young stars are usually much more magnetically active than older ones like the Sun. Vidotto et al. (2014) have proposed to call this link between stellar age and magnetic activity magnetochronology (see also Mathur et al. 2023).

Recently some authors (van Saders et al. 2016; Hall et al. 2021; Metcalfe et al. 2022) have questioned this link between rotation, magnetic activity and age, arguing that passed a certain age (about the age of the Sun for solar twins, i.e 4.5 Gyr), such link is broken, with stellar rotation being "stalled". Others have found that it still holds (Lorenzo-Oliveira et al. 2016, 2019; do Nascimento et al. 2020) with possibly only a temporary pause (Curtis et al. 2020). The difficulty is coming from the large uncertainty of age determination and the observational method used. Supporters of the "stalling" scenario are usually basing their analysis on asteroseismically determined ages and rotation periods calibration using Kepler data. So it is important to also consider theoretical aspects when discussing the relation between age, rotation and magnetic activity levels in the so-called *Sun in time* approach (Ayres 1997; Guinan et al.

2002; Ribas et al. 2005; Güdel 2007; Ahuir et al. 2020; Lorenzo-Oliveira et al. 2020; Johnstone et al. 2021). For instance, one can turn to numerical simulations of solar-like star dynamos to assess trends between various stellar parameters characterizing mass, age, rotation rates or magnetic states. Since the work of Durney & Latour (1977), it is quite clear that the dynamo number  $D$  characterizing the state of the dynamo can be directly linked to the Rossby number  $Ro$  of the star such that  $D \propto 1/Ro^2$ , in classical  $\alpha - \omega$  dynamos. The Rossby number is a key non-dimensional number that is widely used in the study of stellar evolution and activity and can be used to bridge observations and numerical simulations (Brun & Browning 2017; Käpylä et al. 2023). Simply put, it allows quantifying if the turbulence and internal magneto-hydrodynamics in rotating stars is strongly influenced (small Rossby numbers) or not (large Rossby numbers) by rotational effects. The most classical definition of the Rossby number is the so-called *stellar Rossby number*, and is defined as the ratio between a measure of the convective turnover time  $\tau_c$  at a given depth of the convective envelope and the stellar rotation period  $P_{rot}$ , i.e.  $Ro_s = P_{rot}/\tau_c$  (see Landin et al. 2010; Brun et al. 2017 for further discussions on the many definitions of this number). When  $Ro_s$  is small, rotation influences the convection dynamics, tending for very small  $Ro_s$  values to the so-called magnetostrophic state when Lorentz and Coriolis forces (and horizontal pressure gradients) balance one another (Davidson 2014; Augustson et al. 2019). Thanks to an extensive dynamo study published in (Brun et al. 2017, 2022) with the ASH code along with a similar study with the Eulag-MHD code (Strugarek et al.

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2017, 2018), we have now a database of more than 30 fully 3D MHD rotating convective dynamos of solar-type stars, spanning several mass and rotation bins, hence Rossby numbers. In the present paper, we wish to study how the properties of the surface magnetism change as we vary the Rossby number. We are helped by an equivalent systematic observational study of solar-like star magnetism performed by See et al. (2015, 2019b,a) (see also Reiners et al. 2022) in the context of the Bcool consortium (Marsden et al. 2014) using Zeeman Doppler Imaging (ZDI) techniques.

In Sect. 2, we briefly present the set of 15 ASH simulations published in Brun et al. (2022) that we will analyse further in Sect. 3. We conclude in Sect. 4 by proposing a plausible magneto-rotational scenario over secular time scales for solar-type stars.

## 2. Magnetic properties of MHD models of stellar dynamos

### 2.1. Brief overview of stellar dynamo simulation database

In this paper, we focus on the ASH dynamo simulations published in (Brun et al. 2017, 2022). The properties of the various large scale flows and magnetic states, as well as their associated energy and non-linear angular momentum transports in purely HD and MHD conditions, have been characterized in details in these two publications and will not be repeated here. Instead, we wish to focus the analysis in this paper on the global properties of their magnetic field with respect to various stellar parameters.

The simulations represent solar-type stars in a mass range of 0.5 to 1.1  $M_{\odot}$ , rotation rates of 1/8 to 5 times  $\Omega_{\odot}$  at solar metallicity. The effective temperature range considered for the simulations lies approximately between 4030 and 6030 K, covering mostly G and K-type stars on the main sequence.

In all the simulations, the rotating convective envelope of the stars is modelled from the base of their convection zone up to about  $0.97 R_*$ . The range of value covered by the stellar radius is between about 0.44 to 1.23  $R_{\odot}$  and luminosity from about 0.04 to 1.8 times the solar luminosity  $L_{\odot}$ . In the simulations computed with the ASH code, the models also include a stable radiative layer about the same thickness as the convective envelope above, and hence possess a tachocline at the base of their convective zone (in the middle of the computational domain approximately). The diffusivity profiles (viscous, thermal and magnetic) are adapted (tapered) such that they maintain an almost constant Reynolds number throughout the simulations. All these simulations develop a genuine multi-scales convective dynamo, and most have been numerically integrated for several decades of physical time over many years.

From a numerical point of view, the ASH code is a semi implicit pseudo spectral method to solve the anelastic MHD equations in a frame rotating at  $\Omega_*$  (Clune et al. 1999; Brun et al. 2004). Each simulation has a significant stratification, which level depends on the spectral type considered. The radial density contrast varies from about 40 to 80 in the convective envelope and from about 200 to 1000 when including the stable layer. The numerical resolution is  $Nr = 769$  in radius and  $N_{\theta} = 512$  or 1024 in latitude, with  $N_{\phi} = 2N_{\theta}$  (higher horizontal resolution for the low Rossby cases that develop smaller scale dynamics; see Takehiro et al. 2020 for more details on the critical convection mode excitation). We now briefly summarized some of their key magneto-hydrodynamical properties as first analysed in (Brun et al. 2022).

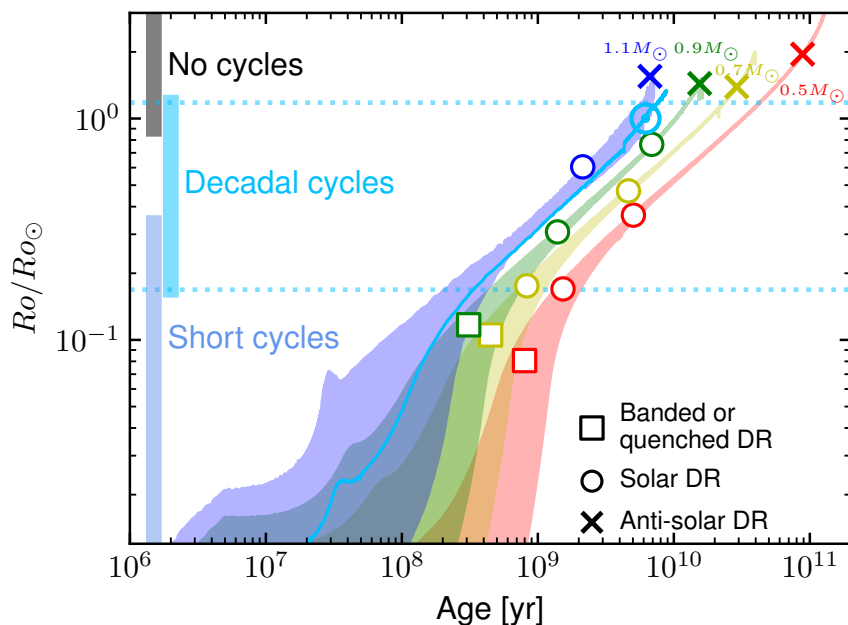
### 2.2. Rotation profiles and their link to magnetic properties: the role of the Rossby number

In Fig. 1 we display the evolutionary tracks of 4 stars having a stellar mass ranging from 0.5 to 1.1  $M_{\odot}$  starting from the PMS all the way to the TAMS, i.e. similar to the mass range used in the 3D MHD dynamo solutions used in this study. To do so, we plot their evolution in a normalized Rossby number vs age diagram, using 1D stellar structure and evolution models computed with the Starevol code (Amard & Matt 2020). We superimposed the 15 ASH models on the plot, to show how our parameter space study can cover several temporal phases of evolution (see also Emeriau-Viard & Brun 2017 for a detailed specific study of the PMS phase). We use the following definition of the Rossby number:

$$Ro_f = |\omega|/2\Omega_* \quad (1)$$

with  $|\omega|$  the mean vorticity of the convective flows, taken at the middle of the convection zone in the ASH simulation and stellar evolution tracks,  $D$  the thickness of the convective envelope and  $\Omega_*$  the model rotation rate. This definition corresponds to the *fluid* Rossby number, a measure of the influence of the Coriolis force on the nonlinear advection term in the Navier-Stokes equation. The fluid and more usually stellar Rossby numbers can be related to one another, as shown in Appendix B (see also Brun et al. 2017 for an overview of the different definitions). In Fig. 1 we normalized it to the value of the Sun  $Ro_{\odot}$  here chosen to be 0.9. Before getting into the details of the figure, we wish to quickly recall how the Rossby number characterizes the dynamics. For low values of the Rossby number, the rotational effects are dominant and force the dynamics to be aligned along the rotation axis (so called Taylor-Proudman constrain; Pedlosky 1987; Brun & Toomre 2002; Miesch et al. 2006; Busse 1983; Busse & Simitev 2006). This usually results into an internal cylindrical differential rotation (DR) profile. For intermediate values, thermal effects via baroclinic torques can bend the iso-contours of  $\Omega$  to be more conical at mid-latitudes, as in the Sun and its helioseismically inferred angular velocity with fast equator and slow poles. For large values of the Rossby number, the rotational effects are weaker and the local angular momentum conservation can lead to anti-solar DR profiles, with slow equator and fast poles. These different states of internal angular velocity profiles are represented by the symbols (cross, square, circle) in Fig. 1.

We can notice several key information of our set of 15 ASH simulations that are plotted as symbols of various shapes and colours. In this stellar evolutionary diagram, we first see a clear trend of stars evolving from the bottom left toward the upper right. Indeed, as stars age they tend to slow down (these 1D stellar evolutionary tracks do not consider a possible stalling of the spin down advocated by some authors as discussed in the introduction and Sects. 2.2 and 3). We note that the large range of Rossby numbers of the 3D MHD dynamo study covers a significant part of the stellar evolutionary track of solar-type stars. Some stars are in the low Rossby number regime (square symbols), whereas others are in the slow rotation regime (cross symbols). We also note that there is a continuous change in the associated DR regimes, going from banded/quenched for early stages toward becoming anti-solar for a long enough secular evolution. As of today, it is difficult to say if solar-type stars will become anti-solar before turning into sub/red giant stars. In Noraz et al. (2022a), we have done a systematic study of the *Kepler* sample published in Santos et al. (2021), and have found 22 possible candidates that would be worth observing further in order



**Fig. 1.** Stellar evolutionary tracks in a Rossby vs age diagram. Shown on the figure are, 1D evolutionary track from Amard & Matt (2020) as a colour shaded envelope taking into account the initial rotational spread. The particular case of the solar track is shown in cyan colour. Symbols represent the ASH simulations and are changed according to the internal differential rotation profile achieved in the convective envelope of the simulations, either banded/quenched, solar like (fast equator slow poles) and anti-solar (slow equator fast poles). The coloured vertical bars on the left represent the magnetic states either short, decadal and no cycles.

to put more constraints on slowly rotating stars. Nevertheless, the continuous transition of states is interesting by itself and could sustain a Sun in time scenario that describe a magneto-rotational dynamical evolution of stars like the Sun over secular ages. Indeed, we also added on Fig. 1, the magnetic dynamo states of the simulations that was found in Brun et al. (2022). For low Rossby number, most models harbour short cycle period dynamo actions. Periods of order half to 2 years are found in the models. For intermediate, more solar-like Rossby number values, corresponding to most of the main sequence of those stars, we find decadal long cycle periods as in the Sun or 18Sco (do Nascimento Jr. et al. 2023). Finally, for evolved old stars, they may lose their cyclic magnetic behaviour and display instead a statistically stationary magnetic states, with very stable polarity in each hemisphere over secular ages before turning into more evolved stages out of the main sequence, for which our 3D MHD study is not designed.

### 2.3. Magnetic butterfly diagrams vs stellar dynamo types

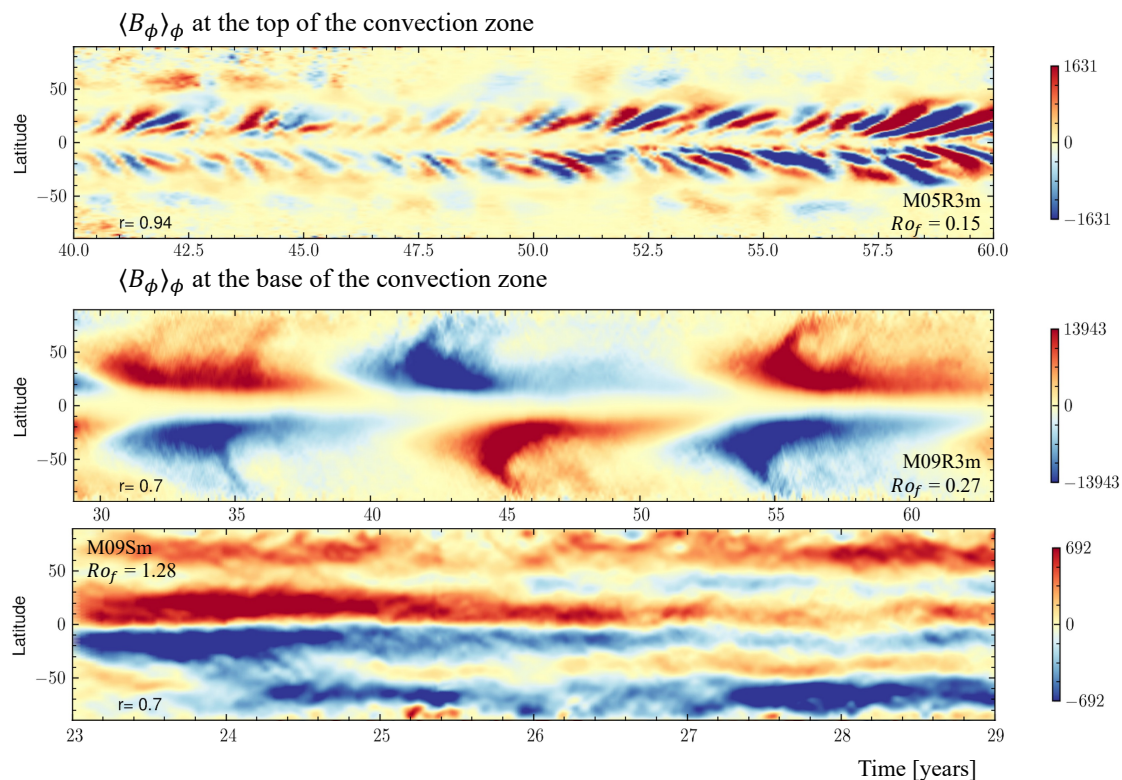
In order to illustrate a bit more the dynamo states achieved in the solar-type stars modelled in the study of Brun et al. (2022), we represent on Fig. 2, three typical magnetic butterfly diagrams found in this 3D MHD parameter study. Butterfly diagrams are formed by azimuthally averaging the toroidal field of the simulation at any depth in the simulation (usually either in the tachocline at the base of the convection zone or near the surface) and by stacking these latitudinal bands in time to form time-latitude contours plots. Each of these diagrams cover several decades of evolution and are strikingly different at first sight.

On the top panel, we show a representative butterfly diagram near the surface for rapidly rotating stars, those with small Rossby numbers. We clearly see the small red/blue alternating colour bands at mid-latitudes, illustrating here local polarity inversions. The short cycle period is of order 6 months in the case illustrated. The bands propagate poleward, and the dynamo wave follows Parker-Yoshimura rule for  $\alpha - \Omega$  dynamo types (as was demonstrated in Brun et al. 2022).

In the middle panel, we show the butterfly diagram near the base of the convection zone for a case with intermediate Rossby number. We clearly see the long cyclic behaviour with 3 consecutive cycles with typical global polarity reversal as seen in the Sun, i.e. the magnetic features have reversed polarity in each hemisphere and the polarity swap signs from one cycle to the next, as Hale et al. (1919) reported from the Sun in his seminal paper. This is due to the dominance of the dipolar/antisymmetric dynamo mode, but some slight out phasing can be seen and this is due to a non-negligible quadrupolar/symmetric mode, that leads to more independent hemispherical magnetic response (Gallet & P etr elis 2009; DeRosa et al. 2012). Another key feature of the middle panel butterfly diagram is the mid-latitude equatorward branch, and a high latitude polar branch. Unlike the fast rotating dynamo cases at low Rossby number, these long period cycle dynamos do not follow the classical Parker-Yoshimura rule. Both  $\alpha$  and  $\Omega$  effects do play a role, but the dynamo loop leading to a cyclic behaviour involves the non-linear retroaction of the large scale toroidal field on the large scale shear (Strugarek et al. 2017). A new cycle starts with the reversed polarity, when the Maxwell stress modifies locally the sign of the gradient  $\partial\Omega/\partial\theta$ . The longer cycle period comes from the time it takes for the field to alter the angular velocity shear, as this occurs only above a certain field strength. Indeed, the  $\Omega$  effect is a linear field stretching mechanism, that takes in the specific case illustrated in Fig. 2 about 10 yr to act. We also find that this dynamo operates much deeper, straddling the base of the convective envelope, where significant energy transfers (up to several % of the solar luminosity) allow global polarity reversals of the large-scale magnetic field in a prey/predator-type mechanism, generating torsional oscillations (see Brun et al. 2022 for their analysis).

In the bottom panel, we also display the butterfly diagram near the base of the convection zone for a typical large Rossby number case ( $Ro_f > 1$ ). This type of dynamo possess an anti-solar DR. As was shown in Noraz et al. (2022b) (see also Karak et al. 2020), such reverse angular velocity profiles (with respect to the Sun) often yields stationary dynamo that do not show clear and systematic polarity reversals. Temporal variability is still present, with the large scale toroidal magnetic wreaths ex-





**Fig. 2.** Typical magnetic butterfly diagrams (i.e. time-latitude diagram of the axisymmetric toroidal magnetic field) achieved in the simulations published in Brun et al. (2022). On the top panel, the butterfly diagram is shown as a colour contour plot in Gauss for the rapidly rotating cases (low Rossby numbers), in the middle panel for intermediate value (Sun-like) and in the bottom panel for statistically steady solutions for slowly rotating stars with large Rossby numbers.

hibiting amplitude variations, sometimes not going all the way around the 360 degree longitudes of the star (see Nelson et al. 2013). Note that here, we do not find any sign of polarity reversal in the non-axisymmetric components of the field, that were found by Viviani et al. (2018).

Our simulations, despite their sophistication, may not faithfully reproduce every nuance of real stellar surface dynamics. Meridional circulation and torsional oscillations amplitudes observed in the Sun, for instance, may not be precisely mirrored in our simulations (see Hotta et al. 2023 for a review). Nevertheless, we do not aim to reproduce the precise solar-case or every detail of solar surface magnetism in this parametric study, but instead wish to unveil broader trends as a function of the Rossby number over evolutionary time-scales. We believe that the trends that will be discussed in Sect. 4 are indicative of genuine energy and force balances occurring within solar-type star convective envelopes (as discussed in Davidson 2013, Aubert et al. 2017, Augustson et al. 2019 and references therein).

In summary, one can thus imagine that as a solar-type star ages, it will respectively go through these 3 magnetic and rotation states. In order to further verify if such stellar magneto-rotational scenario is plausible, we wish to compare other magnetic proxies with recent observations of stellar magnetism. To this end we now turn to search in our dynamo database for various trends with respect to some global stellar parameters by splitting the field in various components (toroidal, poloidal, axisymmetric, dipolar, multipolar, etc...). In the following Sect., 2 simulations out of the set of the 15 simulations presented above (namely M07R3m and M11R5m, see Brun et al. 2022 for naming nomenclature) will not be considered because of a gap in data needed at the time of the present study (spatial and temporal gaps

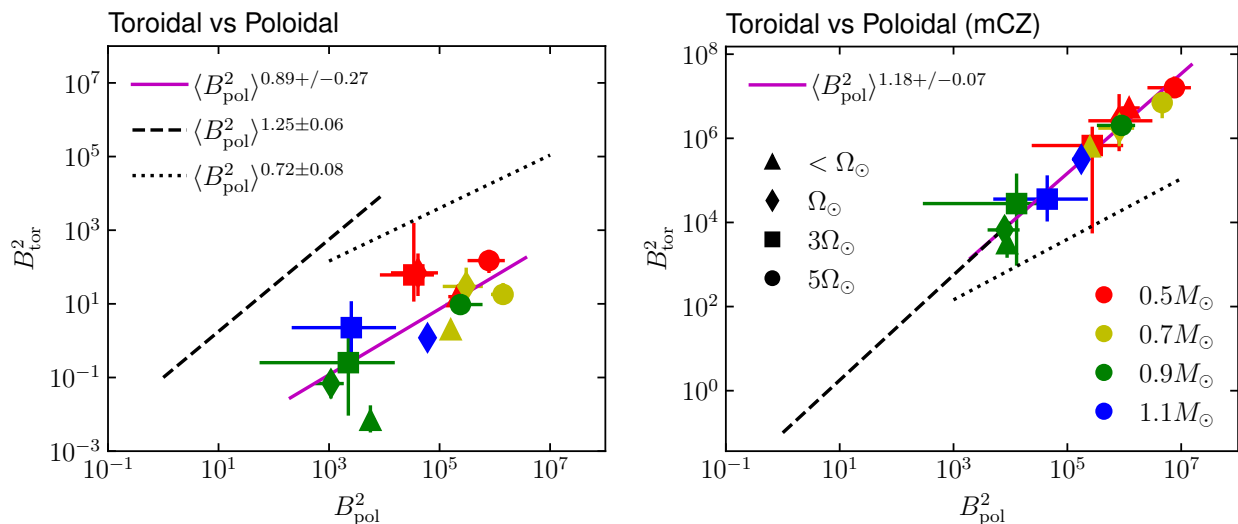
respectively). The top of the numerical domain is  $r_{\text{top}} = 0.95 R_*$  for  $M = 5 M_\odot$  models and  $r_{\text{top}} = 0.97 R_*$  for all the others. Values referred as *near the surface* in the rest of the paper are evaluated at  $r = 0.9997 r_{\text{top}}$  for  $M = 5 M_\odot$  models,  $r = 0.9993 r_{\text{top}}$  for M11R3m, and  $r = 0.9998 r_{\text{top}}$  for all the others.

### 3. Magnetic dynamo trends with stellar parameters

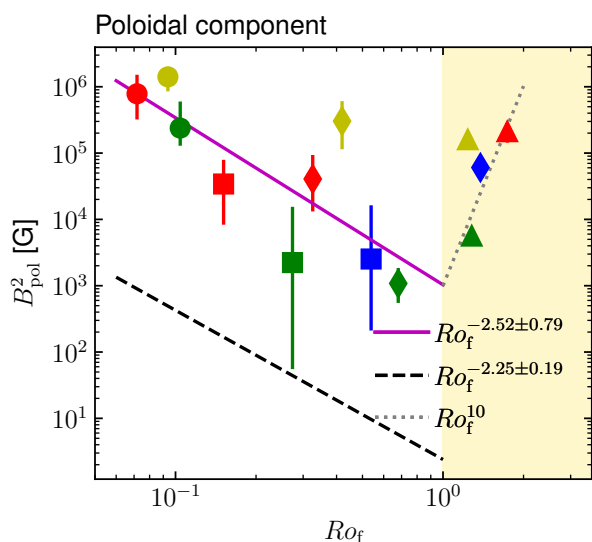
Having recalled the main broad properties of the set of dynamo solutions considered in the present paper (see Brun et al. 2022 for more details), we now wish to look systematically at various trends regarding their magnetic properties with respect to key stellar parameters (such as the Rossby number, stellar mass or field geometry). By doing so we intend to assess how well the set of stellar dynamo simulations can further confirm our Sun in time scenario, by directly comparing our results to those published in the observational studies of See et al. (2015, 2019b,a). To that respect, we will use similar layouts for the figures to ease the direct comparison and plot observational scaling laws (fits) when available in the publications. We account for the difference of Rossby number definition used in observational studies (*stellar*) and the present paper (*fluid*) when showing observational trends as a function of  $Ro_f \approx Ro_s/2.26$  (see Appendix B for more details).

#### 3.1. Poloidal vs Toroidal magnetic field properties

On Fig. 3 we display the magnetic ratio defined here as the relation between the toroidal and the poloidal field amplitude squared. We show this relation near the top of the dynamo so-



**Fig. 3.** *Left:* Magnetic ratio between the squared toroidal  $B_{tor}$  and poloidal  $B_{pol}$  components near the surface. Two observational fits (black dashed for  $M > 0.5M_{\odot}$  and dotted lines for  $M < 0.5M_{\odot}$ ) from See et al. (2015) are indicated, as well as the fit of the simulations (purple line) and their respective error bars. *Right:* Same ratio within the convective envelope. Colour symbols have the following meaning: red, yellow, green and blue represent stars with respectively 0.5, 0.7, 0.9 and 1.1  $M_{\odot}$ , and triangle, diamond, square and filled circle the rotation rates for respectively slow (exact value depends on mass see Brun et al. 2022), one, three and five times the solar rotation rate  $\Omega_{\odot}$ . Error bars indicate min/max values reached by the model during the integrated time (one or several cycle periods) used to compute the mean value indicated by the symbol.

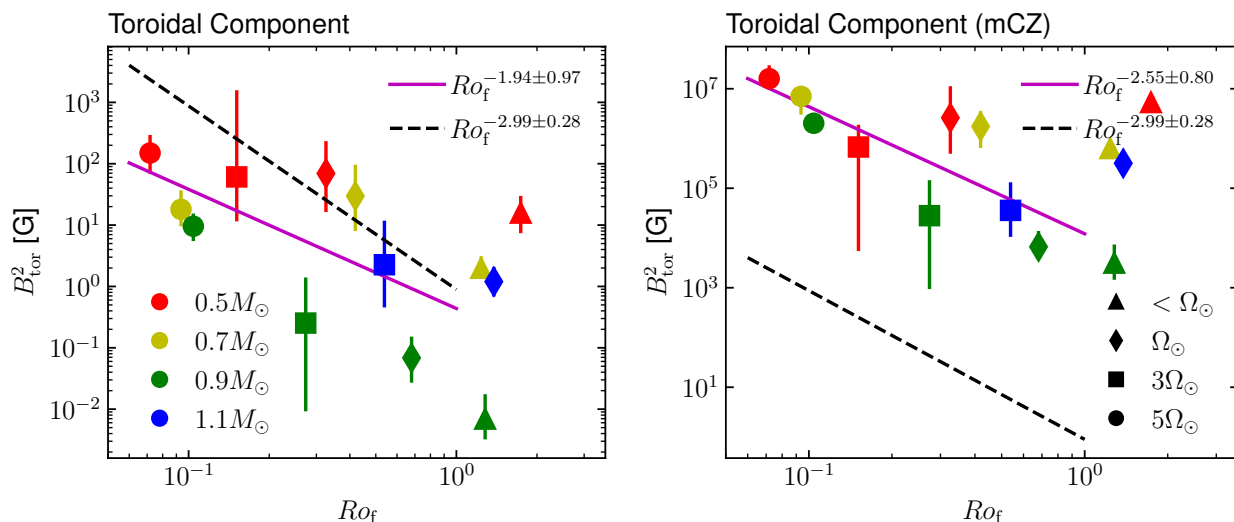


**Fig. 4.** Poloidal field component squared amplitude as a function of the Rossby number  $Ro_f$ . At low  $Ro_f$ , the agreement between the simulations (purple line) and observation (black dashed line) is quantitatively good, with both showing a similar decreasing trend. At higher Rossby number  $Ro_f > 1$  (yellow range) not covered by the observational database an inverse trend is indicated by the simulations, hence suggesting a minimum in poloidal field strength near  $Ro_f \sim 1$ , that could have interesting consequence for stellar spin down via wind braking. An indicative trend proportional to  $Ro_f^{10}$  is shown using a dotted gray line. Note the ‘V-shape’ that the two trends (purple and dotted lines) form, with the minimum being near  $Ro_f \sim 1$ . Colour symbols have the same meaning as in Fig. 3.

lution (left panel) and at the middle of the convective envelope (right panel) for each stellar spectral types. The definition of the poloidal  $B_{pol}$  and toroidal magnetic field  $B_{tor}$  can be found in Ap-

pendix A. To ease direct comparisons to observations, we truncated the spherical harmonics decomposition, conserving only  $\ell \leq 5$  in the computation of both quantities (see Appendix A, and also Vidotto et al. 2016; See et al. 2019b).

Looking first at the result near the surface, we see that the amplitude of the toroidal field is smaller in the set of simulations. We note that the simulation trend  $B_{tor}^2 \propto (B_{pol}^2)^n$  (purple fit) lies in between the 2 values proposed by See et al. (2015), hence capturing the mean trend found in the observational studies. The lower amplitude may be explained by the potential field surface magnetic boundary condition of the simulation, enforcing a zero longitudinally averaged magnetic toroidal field. Even though we probed that interdependence, a few mesh points below the top of the numerical domain (so not quite where the zero value is enforced numerically), this is likely to still have an influence on the ratio between the two field geometries. In order to quantify this effect, we now show on the right panel of Fig. 3 this relation in the middle of the convection zone, where this influence from the boundary condition does not hold any more. We indeed note that the simulations match better the data in terms of amplitude reaching value of  $B_{tor}^2$  around  $10^4 G^2$  and above. The simulation trend matches the higher value of the exponent  $n$  (i.e 1.18 vs 1.25). So this is pretty encouraging that the simulations tend to qualitatively agree with observations. If we further believe the scenario that starspots are created by the surface emergence of flux ropes coming from deeper in the convection zone (as a whole entity or subparts of it), then the observed relation may represent the one of the two field geometries of the deeper interior, as seen here (see also Finley et al. 2024 for further investigations on the M11R3m model of this set). Since in the simulation we do not have yet the formation of large compact magnetic features by lack of local resolution/near surface dynamics, and the choice of potential-field magnetic boundary conditions, the near-surface ratio on the left panel may be biased in the simulation to having much lower near-surface toroidal fields. Hence, assessing this ratio in the middle of the convection zone of the



**Fig. 5.** *Left:* Toroidal field component squared amplitude as a function of the Rossby number  $Ro_f$ , probed near the surface. At low  $Ro_f$ , the agreement between the simulations (purple line) and observational (black dashed line) fits is qualitatively good, with both simulations and observations showing a decreasing trend, although the field amplitude is too low near the surface (left panel). This is likely due to our choice of top magnetic boundary conditions. *Right:* Same quantity probed in the middle of the convection zone, where the agreement is quantitatively good, with both fits being close in terms of power-law index. A possible inverse trend is indicated by the simulations at higher Rossby number  $Ro_f > 1$  (not covered by the observational database), hence suggesting a minimum in the toroidal field strength near  $Ro_f \sim 1$ , that could have interesting consequence for stellar spin down via wind braking.

simulations is less affected by the potential field boundary conditions than at the surface, which is confirmed by comparing the two panels of Fig. 5. Work is in progress to add a realistic atmosphere on top of current global dynamos to have much improved surface magnetic fields boundary conditions (Warnecke et al. 2016; Perri et al. 2021; Delorme et al. 2022; Kaneko et al. 2022).

### 3.2. Trends with the Rossby number

We now turn to considering various trends of the magnetic field and its components with the Rossby number  $Ro_f$ .

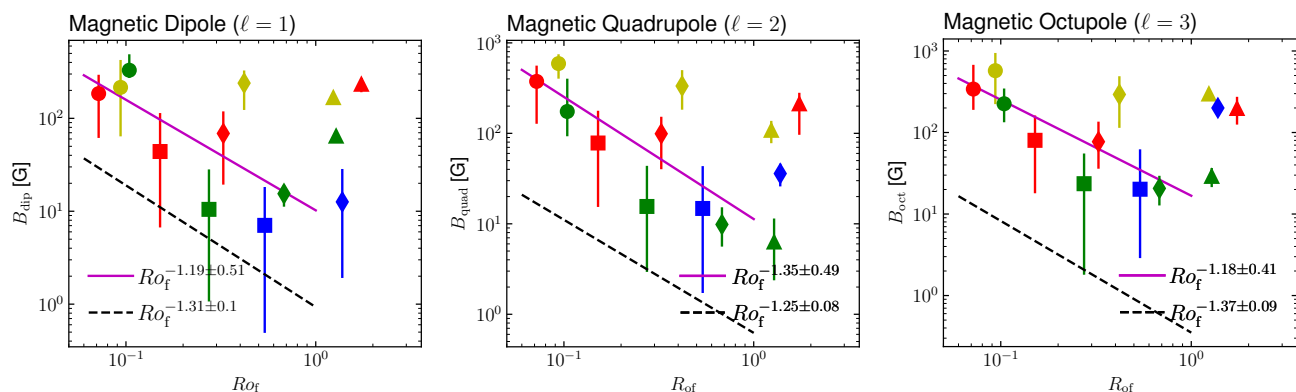
#### 3.2.1. Poloidal and toroidal decomposition

On Fig. 4 we show how the near-surface poloidal magnetic field squared amplitude depends on the Rossby number. At low Rossby number ( $Ro_f < 1$ ) where the observational data are concentrated, the agreement with the observations is quantitatively good regarding the tendency. Another interesting property can be seen on Fig. 4, for large Rossby number values. We see that the trend is opposite in sign, with now  $B_{pol}^2$  increasing with  $Ro_f$  rather than decreasing as for the more rapidly rotating (low  $Ro_f$ ) dynamo cases. This is due to a sharp transition in the DR in the model, going from solar to anti-solar dynamo (see Matt et al. 2011; Gastine et al. 2014; Brun & Browning 2017; Brun et al. 2022). This is a very interesting property that needs to be studied further, as this possible V-shape trend could explain a weaker temporary wind braking, due to a minimum in poloidal field strength as a function of rotation (Rossby number). Brandenburg & Giampapa (2018) and Lehmann et al. (2023) seem also to find a reverse trend for the magnetic flux amplitude of slowly rotating stars in their observational study. Clearly, studying the high Rossby number states is becoming very timely, and obser-

vatinal investigations already started (Noraz et al. 2022a; Donati et al. 2023; Cristofari et al. 2023).

Considering now the toroidal component, we show on Fig. 5 the dependency of  $B_{tor}^2$  with  $Ro_f$  near the surface (left panel) and deeper inside the convection zone (right panel). Near the surface, as for Fig. 2 (left panel), the simulations match only broadly the observed properties. Field values are a bit too low and have only in the large the same declining trend for low  $Ro_f$ . The situation improves significantly when forming the same figure deeper down in the simulation and focusing on the low  $Ro_f$  part of the plot. We see that the simulation trend (purple fit) agrees better with the observations. The field amplitude is a bit too high, indicating that the toroidal field at the stellar surface is likely weaker than deep in the stellar dynamo, but not as weak as the potential field boundary condition imposes (see left panel). We also notice the inverse trend for high  $Ro_f$ , which we will need to look into in the near future (Noraz et al. 2022a). Very little observations are yet reporting toroidal field measurements for high Rossby numbers stars, except for a few M-dwarfs in Donati et al. (2023); Lehmann et al. (2023), not quite directly comparable with our study more focused on G and K-type dwarfs.

Indeed, errors from the simulations fits are not as small as the observations, due to the moderate number of dynamo models compared to the number of observed stars, but overall the fits agree. Please note that running over many decades of physical time these 15 3D MHD global convective dynamo simulation is already a challenge that took several years on massively parallel supercomputers, and that nothing in particular was done or tuned in the simulations to get this comprehensive match with observations. This is reassuring and reinforce our confidence on the set of simulations published in Brun et al. (2022) and further discussed here.



**Fig. 6.** Multipolar decomposition of the magnetic field near the surface (with  $m$  components summed quadratically), showing the dipole  $\ell = 1$  (Left), quadrupole  $\ell = 2$  (Middle) and octupole  $\ell = 3$  (Right) as a function of the Rossby number  $Ro_f$ . At low  $Ro_f$ , the agreement between the simulations (purple line) and observational (black dashed line) fits is quantitatively good for all low degree dynamo modes, with all modes showing a decreasing trend and power-laws comparable to observations.

The possible inverse trend for high Rossby number is also found in all three multipoles. Symbols have the same meaning that in Fig. 5.

### 3.2.2. Multipolar decomposition

It is interesting to further study the properties of the magnetic field of the dynamo simulations by considering the behaviour of single low  $\ell$  spherical harmonics degree magnetic field components such as the dipole, quadrupole and octupole ( $\ell = 1, 2, 3$ ), that can be observed in most ZDI (*Zeeman-Doppler Imaging*) studies of magnetic stars (Petit et al. 2008; Marsden et al. 2014; See et al. 2015, 2019b). We do so in Fig. 6, using the definition listed in Appendix A.

The dipole is the dominant magnetic ingredient for efficient wind braking (then the quadrupole; Réville et al. 2015; Finley & Matt 2017), thus being able to predict its amplitude is key when trying to understand the magneto-rotational evolution of solar-type stars. Looking at the leftmost panel of Fig. 6, we report a good quantitative agreement for the low Rossby number values in terms of trend and amplitude. Again for large values we see that the dipolar field is increasing for slowly rotating stars. This confirms that such a V-shaped dip in the trend could play as a minimum in wind braking efficiency at this Rossby number transition, explaining a possible stalling or weakening of solar-type stars spin down at an intermediate age (van Saders et al. 2016; Curtis et al. 2020).

Turning to the middle and right panels of Fig. 6, we see that both the quadrupolar and octupolar magnetic field components are also in good quantitative agreement with the observational trends. Both purple fits indexes match the observational trends of See et al. (2019a) at low Rossby numbers. They show an inverse trend for large  $Ro_f$  values too, reinforcing the dipolar trend discussed above and explaining why it is also clearly seen in the poloidal component in Fig. 4.

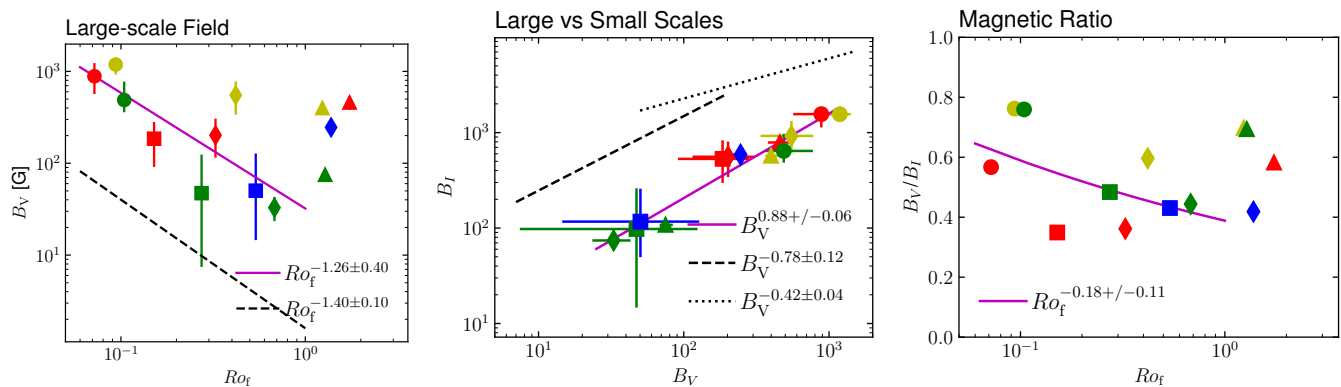
### 3.2.3. Large scale vs total magnetic fields relationships

We now wish to consider how the total magnetic field  $B_I$  vs the large scale field  $B_V$ , obtained by different techniques based on the Zeeman-effect, behave with respect to one another. We already showed in Brun et al. (2022) that filtering the surface magnetic field of the simulations was necessary for a meaningful direct comparison with observations. Indeed, standard equipartition dynamo scaling (Davidson 2014; Augustson et al. 2019),

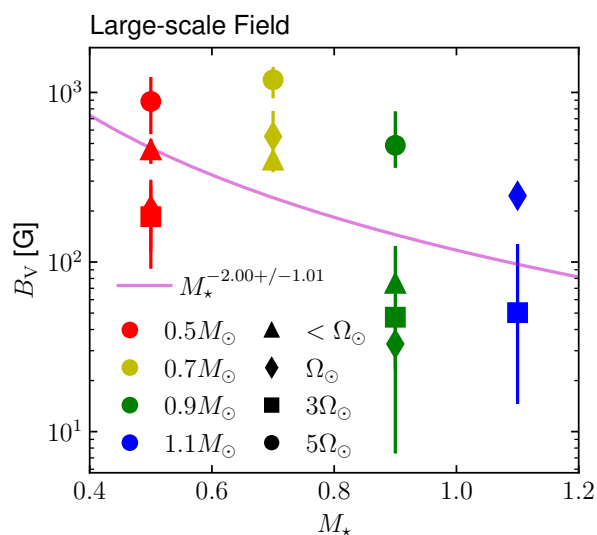
considering the bulk magnetic field, differs from the one derived using surface large scale magnetic field. Comparing the observed  $B_V$  obtained with ZDI techniques with respect to  $B_I$  obtained by Zeeman spectroscopy can help us disentangle the contribution of the large scale vs smaller scale magnetic fields (down to observational limit of distant stars) to the overall dynamo mechanism occurring inside solar-type stars. We define our proxy Stokes V magnetic field  $B_V$  as the filtered low  $\ell$  degree magnetic field (up to  $\ell_{max} = 5$ , see Appendix A and See et al. 2019b). We define our proxy Zeeman total magnetic field  $B_I$  as the normalized near-surface integrated magnetic field keeping all degrees  $\ell$  (see Appendix A). The distinction between these two magnetic field definitions helps to characterize the sensitivity of the large scale field vs the total field to stellar parameter changes. In Fig. 7 we present the dependency of  $B_V$  with  $Ro_f$  (left panel),  $B_V$  vs  $B_I$  (middle panel) and  $B_V/B_I$  vs  $Ro_f$  in the rightmost panel.

On the left panel of Fig. 7 we also report a good quantitative agreement between the simulation and observational fits. We find a decreasing trend for the low Rossby number region of the plot, that matches the observational power-law index of See et al. (2019b) with  $Ro_f$ . We also note the reverse trend for high  $Ro_f$ , as the low degree multipoles shown in Fig. 6 as could be anticipated, given their close relationship. On the middle panel, we plot the total field  $B_I$  as a function of  $B_V$ , in a way similar to what we did in Fig. 3 for the poloidal and toroidal components. The trend is clear, both fields are positively correlated and not quite linearly related. We also note that  $B_V$  is larger and  $B_I$  smaller than typical amplitude expected from observations, both near the surface and deeper in the convection zone (the latter is not shown here). This is likely to be a limitation of the LES (*Large-Eddy-Simulation*) approach adopted in these simulations, where the dynamics of the smallest scales is not resolved. Nevertheless, the trends we find are robust, implying that the magnetic field energy distribution (spectrum) between large and small scales fields is expected to evolve as the star ages. In that context, we finally show in the rightmost panel how the ratio  $B_V/B_I$  behaves with respect to the Rossby number. This is to verify if, as the star rotate slower and slower, there is a tendency to build less large scales magnetic fields. There is a weak trend, stating that indeed the large fields tend to diminish slightly faster than the total field, implying a busier and smaller scale magnetic





**Fig. 7.** Various trends for our theoretical proxies of both the Stokes V magnetic field measurement,  $B_V$ , and total field measurement via Zeeman spectroscopy,  $B_I$ . *Left:* The  $B_V$  dependency with Rossby number is well captured, both fits (purple for the simulations and black dashed lines for the observations) agreeing quantitatively well in terms of power-law index. *Middle:* Magnetic relation between  $B_V$  and  $B_I$  showed as in Fig. 3, showing an expected positive trend. *Right:*  $B_V/B_I$  ratio as a function of  $Ro_f$ , with a fit (purple line) indicating a weak dependency. Symbols have the same meaning that in Fig. 8.



**Fig. 8.** Theoretical proxy for Stokes V magnetic field  $B_V$  vs stellar mass in the simulations data set. Symbols have the same meaning as on Fig. 3. We note a rather strong decreasing trend with stellar mass (purple fit exponent around -2), the rotation rate leading to some spread and the relatively large error bar on the fit.

field near stellar surfaces. Such a broad tendency can be seen in observational data of See et al. (2019b) (see their Figure 2) for  $M > 0.5M_\odot$ , however the number of stars considered is also small, and any stronger conclusion will need further investigations on both theoretical and observational sides. Again, when  $Ro_f$  goes over 1 and the differential rotation of the star flips direction, harbouring now slow equator and fast poles, the situation reverses and stronger large scale magnetic fields are generated by the dynamo. Hence, there is still a peculiar region both in field geometry and amplitude near  $Ro_f \sim 1$ , that we will investigate in the near future both observationally and theoretically.

On Fig. 8, we display how the large scale magnetic field  $B_V$  scales as a function of stellar mass. In the study, we cover 4 mass bins that can help us search for possible trends. We see that indeed there is a decreasing field amplitude with stellar mass, with the fit indicating a high negative exponent -2, but

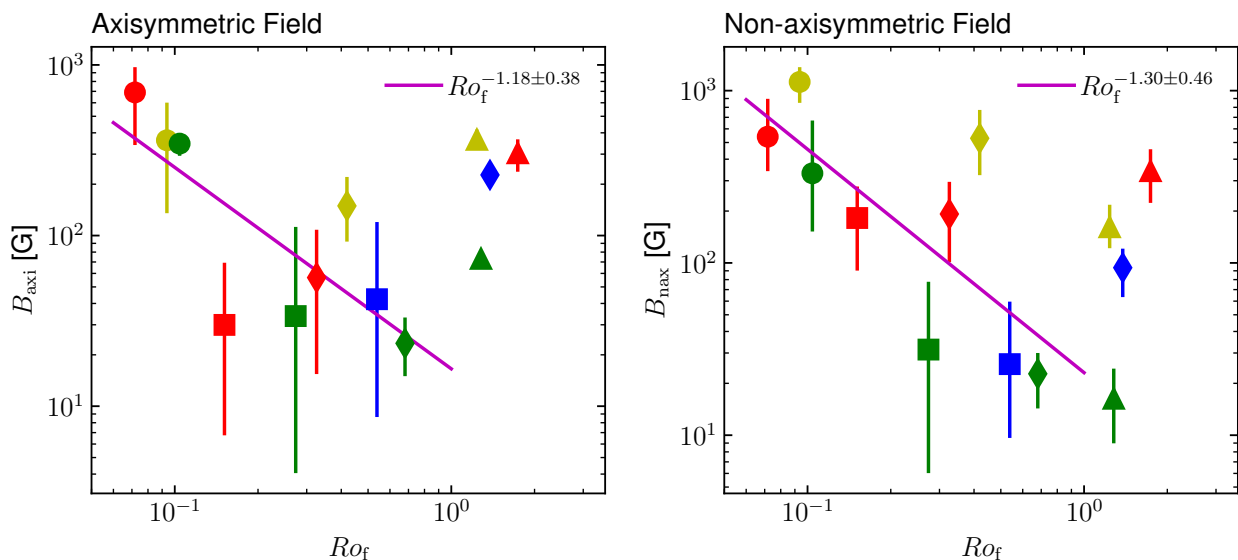
with a relatively large error bar due to some rotational spreading. This result could seem counterintuitive, as more massive stars are more luminous (recall that for solar-type stars  $L_* \propto M_*^4$ , Hansen et al. 1995). However, this effect is compensated by the fast narrowing of the convective envelope with stellar mass, resulting into a much lower averaged density in the convective envelope, hence a lower kinetic energy reservoir (Brun et al. 2017, 2022). Observations by Johns-Krull & Valenti (2000) also found that the equilibrium field  $B_{eq}$  in M dwarfs possesses a higher amplitude when compared to that of G or F stars. They use the definition of the equilibrium field as being that satisfying  $P_{mag} \sim B_{eq}^2 = P_{gaz} \sim \bar{\rho} T_{eff}$  at the surface (Brun et al. 2015), and they found a relatively good agreement with their measurements, for the same reason, that  $T_{eff}$  varies less than the mean density  $\bar{\rho}$ , when going from stellar spectral M to F.

### 3.2.4. Axi- vs non-axisymmetric magnetic field trends

On Fig. 9 we are showing how the axisymmetric ( $m = 0$ ; left panel) and non-axisymmetric ( $m \neq 0$ ; right panel) magnetic field components are behaving with respect to the Rossby number. We wish to see if the field tends to be less regular and axisymmetric under some conditions.

In the low Rossby number range, we note that the field geometry tends to keep its axi- vs non-axisymmetric nature. There is a slightly larger exponent of the simulations fit for the non-axisymmetric magnetic field component, although great care has to be kept in mind regarding the relevance of such a power-law fit in the case of the axisymmetric field near the surface for low  $Ro_f$ . This tends to indicate that the rotation near  $Ro_f \sim 1$  favours relatively more regular axisymmetric magnetic fields in the simulations. This trend is more pronounced for larger values of  $Ro_f$ . We clearly see that axisymmetric field is particularly enhanced for the faster rotating cases ( $5\Omega_\odot$ , circle shapes). We also note that the triangle symbols, that again present a reverse trends, are higher in amplitude in the axisymmetric part, hence indicating a more symmetric magnetic field. This can be easily understood by returning to Fig. 2 bottom panel, where we display a butterfly diagram for a typical slowly rotating case. We see that these type of models, possessing anti-solar DR, usually develop large scale statistically steady magnetic wreaths in both hemispheres. Such wreaths are intricate and intertwined large scale magnetic field





**Fig. 9.** Axisymmetric ( $m = 0$ , *Left*) and non-axisymmetric ( $m \neq 0$ , *Right*) magnetic field decomposition variations with the Rossby number. No clear differences are seen between the two field components, the purple fit exponents in the low Rossby number regions being close and negative, with at best a tendency for the field to be more axisymmetric for larger Rossby number values. An inverse trend is also seen for large Rossby number, as expected from the previous analysis. Symbols have the same meaning that in Fig. 8.

ribbons, that can be stable over relatively long periods of time (Brown et al. 2010). They are dominantly axisymmetric, even though there are known to have their own complex dynamics as the degree of turbulence of the simulation is increased (Brown et al. 2008; Nelson et al. 2013).

#### 4. Discussion and conclusion

We have discussed how well a set of 15 3D MHD global dynamo solutions compare to stellar magnetism observations of solar-type stars. Overall, we find a very good quantitative agreement between our set of 15 simulations and the observational studies from the Bcool consortium (Marsden et al. 2014) published in See et al. (2015, 2019b,a). From this ensemble study, several interesting features were found. For instance, we find that for low Rossby number values  $Ro_f < 1$ , the various trends in the range  $Ro_f = [0.05, 1]$  imply a decreasing magnetic field amplitude with a relatively steep slope. The poloidal, toroidal or multipolar decompositions all follow a relatively clear decreasing trends with  $Ro_f$ , confirming that young stars rotating fast tend to have a larger field amplitude than their older counterparts. We also noted that the near surface toroidal magnetic field amplitude is too low in the simulations. We believe this is due to our choice of potential field magnetic boundary condition that sets the toroidal field to zero at the top of the numerical domain. When comparing the toroidal magnetic field strength deeper down in the simulations, we recover a better agreement with observations, both in terms of magnetic field amplitude and sensitivity to the Rossby number. This somewhat confirms that there is a clear link between the dynamo mechanism, operating deep in the convective envelope and in the tachocline, that generates the toroidal field and leads to the subsequent emergence of magnetic fields on the surface of the observed solar-type stars.

An interesting result to keep in mind is a possible reversal of the magnetic trend when reaching higher Rossby numbers  $Ro_f > 1$ , pointing then toward an enhancement of the stellar magnetism as the rotational influence is decreasing. This

change of trend suggests a minimum of large-scale field amplitude around  $Ro_f \sim 1$ , forming a "V-shape", that could have interesting consequence for stellar spin down via wind braking. If such a behaviour is indeed occurring in stellar dynamos, then stars reaching this rotational state could potentially be in a minimum state of wind braking (due to the local weakening of the large-scale magnetic field (dipole) in the parameter space). This in turn could explain some observational claims advocating for a stalling scenario of stellar spin down (van Saders et al. 2016; Curtis et al. 2020; Hall et al. 2021), deviating from classical gyrochronology rotational laws (Skumanich 1972; Barnes 2003). Some hints that a reverse trend of the magnetic flux for slow rotators may exist have been put forward in Brandenburg & Gimpapa (2018). While the existence of a minimum in field amplitude near  $Ro_f \sim 1$  is clear in our study, this does not lead to a full stop of the stellar spin down process, but rather to a significant slowing down of the spin down process, that could then be revived once the star has slowly but surely crossed the  $Ro_f > 1$  transition. Such a deceleration of the rotational evolution could be further amplified if a decrease of the mass-loss rate is also happening (Metcalf et al. 2022). However, observational constraints of the high Rossby range  $Ro_f > 1$  is still limited as the sensitivity of current observational techniques decreases drastically for slow-rotations (Donati et al. 2006; Benomar et al. 2018), and the possible disappearance of starspots due to a change in the dynamo nature would further make observational characterization of high Rossby targets difficult with photometric techniques. In that context, more study of stellar magnetism for stars near the  $Ro_f = 1$  transition must be undertaken to confirm that a transition exists. In Noraz et al. (2022a), we have proposed a list of slowly rotating (probably anti-solar) stellar candidates, and we hope to be able to study their magnetic properties in the near future and confront it to our stellar magneto-rotational scenario. On that aspect, first large-scale magnetic field quantifications for high Rossby M-dwarfs were recently reported (Donati et al. 2023; Lehmann et al. 2023), and

we expect to see it soon for solar-like G-K stars modelled in the present paper.

While very encouraging, this study can be improved along several directions. For instance, the Reynolds number of the dynamo simulations presented in this study have low to intermediate values (see Brun et al. 2022, Table 2), and as such are only numerical experiments to try to understand the complex and highly nonlinear nature of stellar dynamos. Each of the individual simulations could possibly be an approximate realization of the real single star it is supposed to represent. Indeed, global simulations of rotating convection still struggle to perfectly reproduce the solar case. In particular, there is currently a mismatch between global convection simulations and helioseismic observations regarding the power contained in giant convection cells, known as the “convective conundrum” (O’Mara et al. 2016; Hotta et al. 2023). This results in an effective Rossby number slightly too large in global convection simulation for the solar rotation rate. In this context, relative comparisons between models with different Rossby numbers can be done, but the absolute positioning of a given solar-type star should be considered with care. However, the comprehensive agreement found between Brun et al. (2022) and Strugarek et al. (2017), using intrinsically different numerical methods, makes us confident about the relevance of these dynamo solutions to explore and discuss the physical nature of solar-like cyclic activity, and the robustness of the overall trends found in our study. It is now important to reiterate that our primary focus is on identifying and elucidating overarching trends. We intentionally avoided any fine-tuning of our simulations to force agreement with every observational detail. We thus find particularly encouraging that such overall trends agreements arise without ad hoc adjustments. The existence in the simulation data of a magnetic upsurge in the high-Rossby number regime then becomes a promising avenue for future research. For instance, the possibility of a significant change in the dynamo process in this high Rossby number regime will need further investigation (Noraz et al. 2022a; Donati et al. 2023; Cristofari et al. 2023; Lehmann et al. 2023).

This relatively successful ab initio approach is encouraging for future studies that will introduce higher degree of turbulence (Reynolds numbers) and more realistic surface boundary conditions (Perri et al. 2021; Delorme et al. 2022; Kaneko et al. 2022). To summarize, this study suggests that a coherent magneto-rotational scenario for solar-type stars over secular evolution time, as summarized in Fig. 1 and in Sects. 2.2 and 3, is plausible. That is, the young stars start rotating relatively fast and are very active with short period magnetic cycles. Then as they age and reach intermediate Rossby number values, their cycle period increases, reaching decade long time spans due to a change of the dynamo operation. Near the  $Ro_f \sim 1$  transition, the stars may undergo a weakening of their wind braking, with what could appear like a stalling. As the stars age and continue to spin down more slowly, they may reach high Rossby number values and reverse their angular velocity profile. This new state of internal rotation modifies the stellar dynamo once more, leading possibly to a loss of its cyclic behaviour and the building of stronger large scale magnetic fields, resulting in a revival of stellar spin down.

*Acknowledgements.* All authors are thankful to L. Amard for providing the 1D stellar evolution models used to make Fig 1 and to V. See and P. Petit for useful discussions regarding the Bcool consortium observations of stellar magnetism. We acknowledge GENCI project 1623 for access to supercomputing infrastructures where most of these 15 simulations were computed. We acknowledge funding support by ERC Whole Sun #810218, INSU/PNST grant and Solar Orbiter and Plato CNES financial supports.

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## Appendix A: Magnetic field decomposition definitions

In order to compare our simulations with the published results and figures of See et al. (2015, 2019b,a), we will use the formalism of Donati et al. (2006); Vidotto et al. (2016); Folsom et al. (2018) (see also Rieutord 1987). In this formalism, we make use of the vectorial spherical harmonics basis defined as

$$\begin{cases} \mathbf{R}_l^m &= Y_l^m \mathbf{e}_r \\ \mathbf{S}_l^m &= \nabla_{\perp} Y_l^m = \partial_{\theta} Y_l^m \mathbf{e}_{\theta} + \frac{1}{\sin\theta} \partial_{\varphi} Y_l^m \mathbf{e}_{\varphi} \\ \mathbf{T}_l^m &= \nabla_{\perp} \times \mathbf{R}_l^m = \frac{1}{\sin\theta} \partial_{\varphi} Y_l^m \mathbf{e}_{\theta} - \partial_{\theta} Y_l^m \mathbf{e}_{\varphi} \end{cases}, \quad (\text{A.1})$$

where  $Y_l^m$  are the orthonormalized classical spherical harmonics (defined such as  $\int Y_{l_1}^{m_1} (Y_{l_2}^{m_2})^* d\Omega = \delta_{l_1, l_2} \delta_{m_1, m_2}$ ). The vector spherical harmonics basis A.1 can be used to decompose any vector  $\mathbf{B}$  such that,

$$\mathbf{B} = \sum_{\substack{\ell, m \\ \ell \leq \ell_{\text{cut}} \\ -\ell \leq m \leq \ell}} \mathcal{A}_{\ell, m} \mathbf{R}_l^m + \mathcal{B}_{\ell, m} \mathbf{S}_l^m + \mathcal{C}_{\ell, m} \mathbf{T}_l^m. \quad (\text{A.2})$$

Based on this decomposition and for direct comparison to observational results, we use the following definitions of the various magnetic field components in the present paper

$$B_I^2 = B_{\text{rms}}^2 = \frac{1}{4\pi} \sum_{\substack{\ell, m \\ \ell \leq \ell_{\text{max}} \\ -\ell \leq m \leq \ell}} |\mathcal{A}_{\ell, m}|^2 + \ell(\ell+1)(|\mathcal{B}_{\ell, m}|^2 + |\mathcal{C}_{\ell, m}|^2), \quad (\text{A.3})$$

$$B_V^2 = \frac{1}{4\pi} \sum_{\substack{\ell, m \\ \ell \leq \ell_{\text{cut}} \\ -\ell \leq m \leq \ell}} |\mathcal{A}_{\ell, m}|^2 + \ell(\ell+1)(|\mathcal{B}_{\ell, m}|^2 + |\mathcal{C}_{\ell, m}|^2), \quad (\text{A.4})$$

$$B_{\text{pol}}^2 = \frac{1}{4\pi} \sum_{\substack{\ell, m \\ \ell \leq \ell_{\text{cut}} \\ -\ell \leq m \leq \ell}} |\mathcal{A}_{\ell, m}|^2 + \ell(\ell+1)|\mathcal{B}_{\ell, m}|^2, \quad (\text{A.5})$$

$$B_{\text{[dip;quad;oct]}}^2 = \frac{1}{4\pi} \sum_{\substack{\ell, m \\ \ell \in \{1,2,3\} \\ -\ell \leq m \leq \ell}} |\mathcal{A}_{\ell, m}|^2 + \ell(\ell+1)|\mathcal{B}_{\ell, m}|^2, \quad (\text{A.6})$$

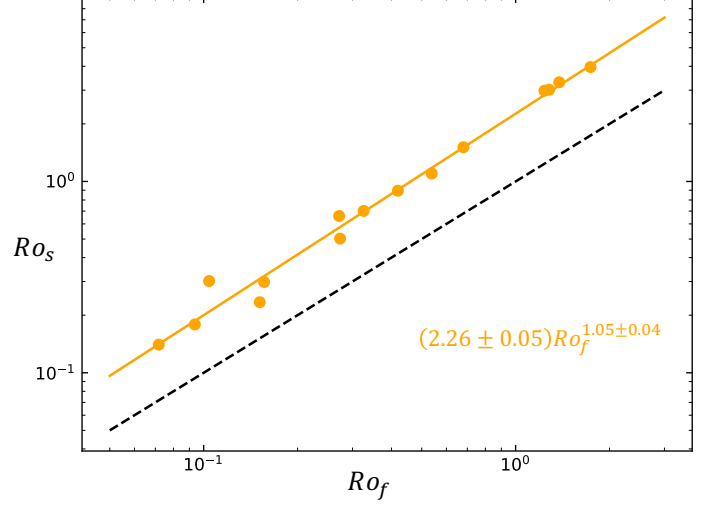
$$B_{\text{tor}}^2 = \frac{1}{4\pi} \sum_{\substack{\ell, m \\ \ell \leq \ell_{\text{cut}} \\ -\ell \leq m \leq \ell}} \ell(\ell+1)|\mathcal{C}_{\ell, m}|^2, \quad (\text{A.7})$$

where  $\ell_{\text{cut}} = 5$  and  $\ell_{\text{max}} = 2N_{\theta}/3$ . The numerical module *SHTns* (Schaeffer 2013) has been used here to compute the different decompositions.

## Appendix B: Relation between the stellar and the fluid Rossby number

In the present paper, we decided to adopt the *fluid* definition of the Rossby number  $Ro_f$  as it is the direct quantification of the advection term over the Coriolis term in the momentum equation of fluid dynamics, that we used in the previous study Brun et al. (2022) to characterize the different rotational and magnetic states

of the set of simulations presented in Sect. 2. However, this number is not directly accessible from observations (see Noraz et al. 2022a), and the observational trends we refer to in the present paper were computed with the *stellar* definition of the Rossby number  $Ro_s$  (See et al. 2015, 2019b,a). In order to compare fairly our simulations to observations, we illustrate on Fig. B.1 the relationship between both definition in our set of models.



**Fig. B.1.** Comparison of the *stellar* Rossby number  $Ro_s$ , as a function of the *fluid* one  $Ro_f$ , in the different models (orange dots) from Brun et al. (2022). A linear regression is proposed with the solid orange line ( $Ro_s = (2.26 \pm 0.05) Ro_f^{1.05 \pm 0.04}$ ), along with the direct proportionality  $Ro_s = Ro_f$  illustrated with the dashed black line.

The fluid Rossby number  $Ro_f$  is computed following Eq. 1 and taken in the middle of the convection zone. The stellar Rossby number  $Ro_{\text{rms}} = P_{\text{rot}}/\tau_c^{\text{CS}}$  is computed similarly to See et al. (2015, 2019b), considering the empirical expression of the convective turnover time derived by Cranmer & Saar (2011), which is  $\tau_c^{\text{CS}} = 314.24 \exp[-\frac{T_{\text{eff}}}{1952.5 \text{ K}} - (\frac{T_{\text{eff}}}{6250 \text{ K}})^{18}] + 0.002$  days. We see that the relationship between both definitions is close to be linear, with a proportionality factor  $Ro_s \simeq 2.26 Ro_f$ . An obvious factor 2 comes from their respective definitions, making the fluid Rossby number smaller, the reminder coming from small numerical differences. We use this calibration factor when plotting the observational trends as a function of the fluid Rossby number  $Ro_f$  in Figs. 4, 5, 6 and 7.

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